Truth Tellers

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A truth-teller sentence is sentence that states its own truth. Unlike liar sentences, truth tellers can be be given any truth value without engendering paradox in a setting where the truth predicate is treated as a new primitive predicate. In Kripke's (1975) theory, for instance, truth-teller sentences come out as *ungrounded*; they lack a truth value in certain fixed-point models, are true in certain other fixed-point models and false in still others. Many authors have argued that truth tellers should be treated as semantically defective and not be assigned a fixed classical truth value.

Unlike liar sentences, truth-teller sentences can be constructed in a purely arithmetical setting, where truth (restricted to sentences of a certain restricted class) is defined by an arithmetical formula. For instance, one can specify a Σ_1 formula Tr that expresses (in some sense) the property of being a true Σ_1-sentence. Applying the diagonal lemma to Tr, yields a Σ_1-sentence τ that states its own Σ_1-truth. This sentence is purely arithmetical and therefore is either true or false in the standard model of arithmetic.

As may be conjectured, whether the Σ_1-truth teller is true or false, provable, refutable or independent, depends on subtle choices in defining it, as McGee observed some time ago (unpublished). In (Halbach and Visser, 20??) it is shown that the properties of truth-teller sentences by varying the coding scheme, the formula expressing truth and the way self-reference is obtained.

The results can be extended to other complexity classes, that is, to Σ_n ($n > 1$) and Π_n ($n > 0$).

The results provide plenty of examples for intensionality in metamathematics. I shall discuss various aspects of this intensionality. Moreover, as is shown by various constructions, the notion of self-referentiality in arithmetic is much less clear than one may expect.

References

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