

*Matrix Tricks for Linear Statistical Models:  
Our Personal Top Twenty*

Updates in the References<sup>1</sup>

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♠ **New References, Likely to Be Added (in Due Course ...)**

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