Rank-Based Inference on the Shape of Elliptical Distributions

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Abstract

We propose (i) a class of rank-based procedures for testing that the shape matrix of an elliptical distribution is equal to some fixed value (this problem includes the problem of testing for sphericity as a particular case), as well as (ii) a class of R-estimators for the shape parameter. The proposed tests/estimators are invariant/equivariant under translations, monotone radial transformations, rotations, and reflections with respect to the estimated center of symmetry. For adequately chosen scores, they are locally asymptotically optimal (in the Le Cam sense) at given densities. The multivariate ranks used throughout are those of the distancesin the metric associated with the null value of the shape matrix (for testing problems) or with a preliminary estimate of the shape parameter (for the estimation problem)between the observations and the estimated center of the distribution. Asymptotic relative efficiencies with respect to the standard Gaussian procedures (i.e., pseudo-Gaussian LRT and MLE) are derived, and are shown to be uniformly high for specific choices of the score functions. The proposed tests are valid without any moment assumption. As for the proposed R-estimators, they are defined as iterative M-estimators. Unlike those obtained via the methods described in Marc Hallins talk, these do not require the difficult estimation of a cross-information coefficient. Nevertheless, they are root-n consistent only under a (very) mild condition on this unknown cross-information coefficient. We also compute their influence functions and show that, similarly as for univariate R-estimation for location, a broad range of robustness behaviors can be obtained by considering various types of score functions.