The Spatial Multivariate Quantile Function: Strengths, Weaknesses, Competitors

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Abstract

In recent years a number of quite different multivariate depth and quantile functions have been formulated and investigated. A timely question is whether one of these can or should be adopted as the best choice for practical use. This talk will focus on the spatial multivariate quantile function (Choudhuri, Dudley) as a benchmark. For this case, we first will review basic features and characterizations and associated depth, centered rank, and outlyingness functions. We then will treat productive results now available for this quantile function: the influence function, masking and swamping breakdown points of associated outlier identification procedures, cluster analysis procedures, multivariate sign test procedures, and a Bahadur-Kiefer (B-K) representation. We also will introduce an extension to a spatial U-quantile function, along with an extended B-K theorem. For the empirical spatial U-quantile function, the B-K representation provides a useful U-statistic approximation. In terms of spatial U-quantiles, interesting new multivariate nonparametric estimators and test statistics can be formulated, for example generalized multivariate signed-rank tests, an extension to multiple regression of Theil's nonparametric simple linear regression slope estimator, and a new matrix-valued dispersion measure whose sample analogue estimator has breakdown point 0.293 independently of the dimension of the data. We will also discuss equivariance limitations of the spatial quantile function and explore whether these can be overcome by a suitable modification. Finally, we will examine major competing depth and quantile approaches comparatively against the strengths and weaknesses of the spatial quantile function.