Graphical Comparison of Multivariate Nonparametric Location Tests for Restricted Alternatives

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Outline

1. Restricted Location Alternatives in $\mathbb{R}^p$
2. Composite Null Hypotheses
3. Graphical Comparison Methods

1. Restricted Location Alternatives in $\mathbb{R}^p$

Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be i. i. d. random vectors in $\mathbb{R}^p$ with distribution F, where F has a symmetry center $\boldsymbol{\vartheta}$.

Univariate Case

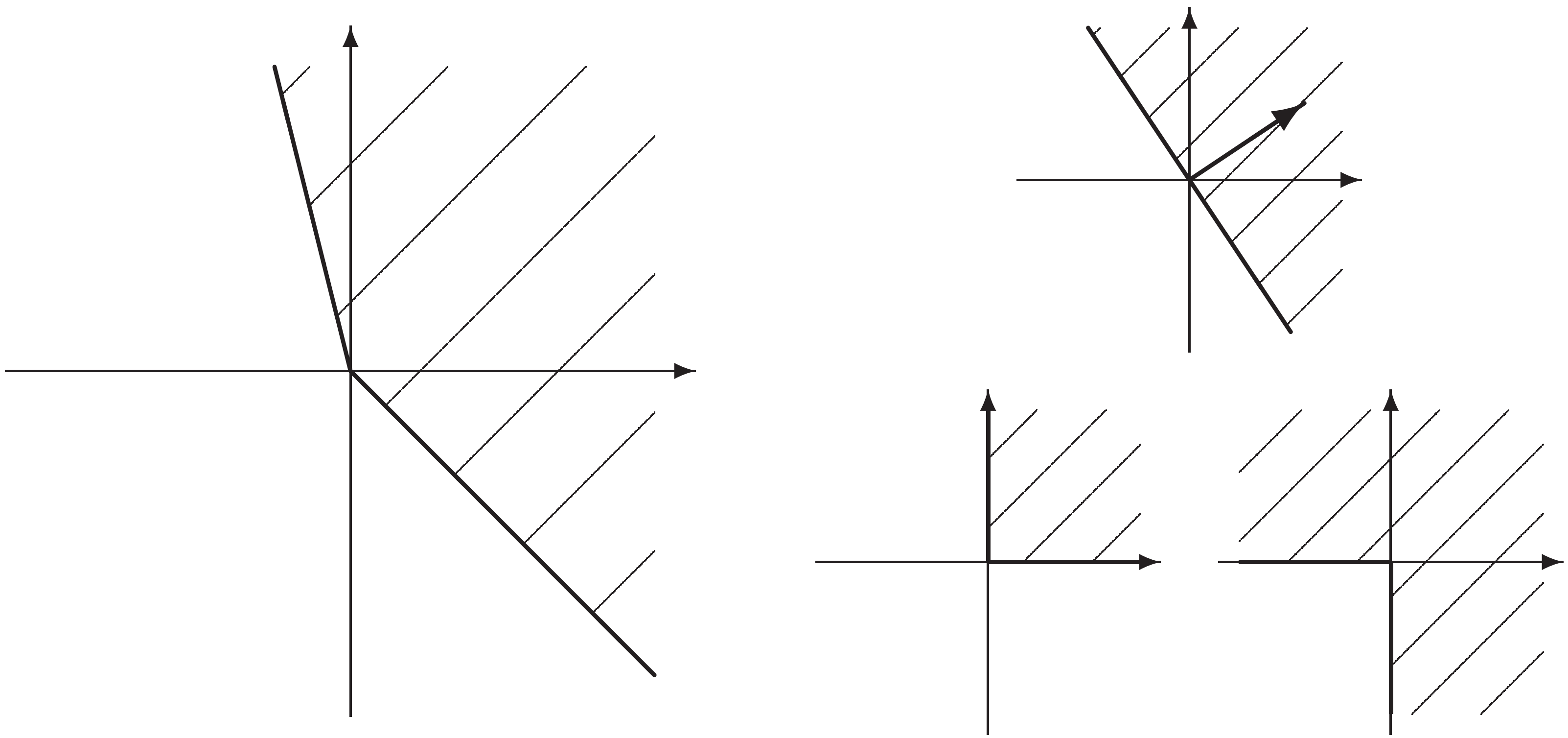
$$H_0 : \vartheta = 0 \quad \text{vs.} \quad H_1 : \vartheta > 0$$

Multivariate Case

Specification of a set Θ_1 with $\mathbf{0} \in \partial\Theta_1 \setminus \Theta_1$.

$$H_0 : \ \boldsymbol{\vartheta} = \mathbf{0} \quad \text{vs.} \quad H_1 : \ \boldsymbol{\vartheta} \in \Theta_1$$

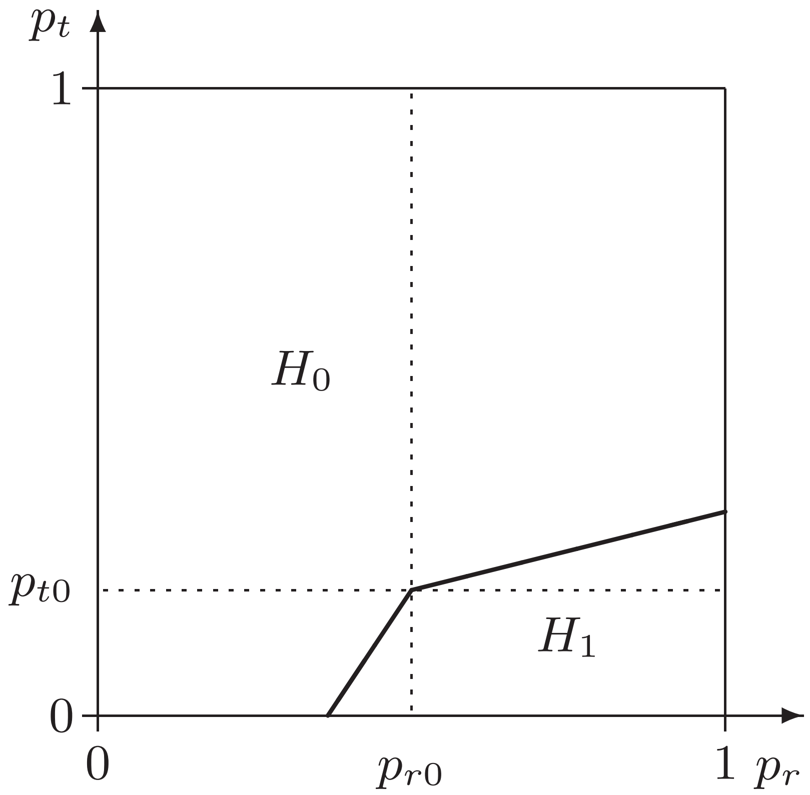
Θ_1 can e. g. be a cone or a sector in $\mathbb{R}^2$:



Example of a Sector Alternative Application

Conaway and Petroni (1996):

Trade-off between response and toxicity in tumor therapy



2. Composite Null Hypotheses

Univariate case: The usual tests for

$$H_0:\ \vartheta = 0 \quad \text{vs.} \quad H_1:\ \vartheta > 0$$

also respect the level if they are applied to

$$H_0:\ \vartheta \leq 0 \quad \text{vs.} \quad H_1:\ \vartheta > 0.$$

Multivariate case:

Θ_1 complement of a non-convex set $\Rightarrow$ distinction is important.

Example:

$$H_0:\ \exists\, i \in \{1,\ldots,p\} : \vartheta_i \leq 0 \quad \text{vs.} \quad H_1:\ \vartheta_i > 0,\ \forall\, i \in \{1,\ldots,p\}$$

Tests with critical values based on $\boldsymbol{\vartheta} = \mathbf{0}$ only:
Usually liberal at the orthant border.

3. Graphical Comparison Methods

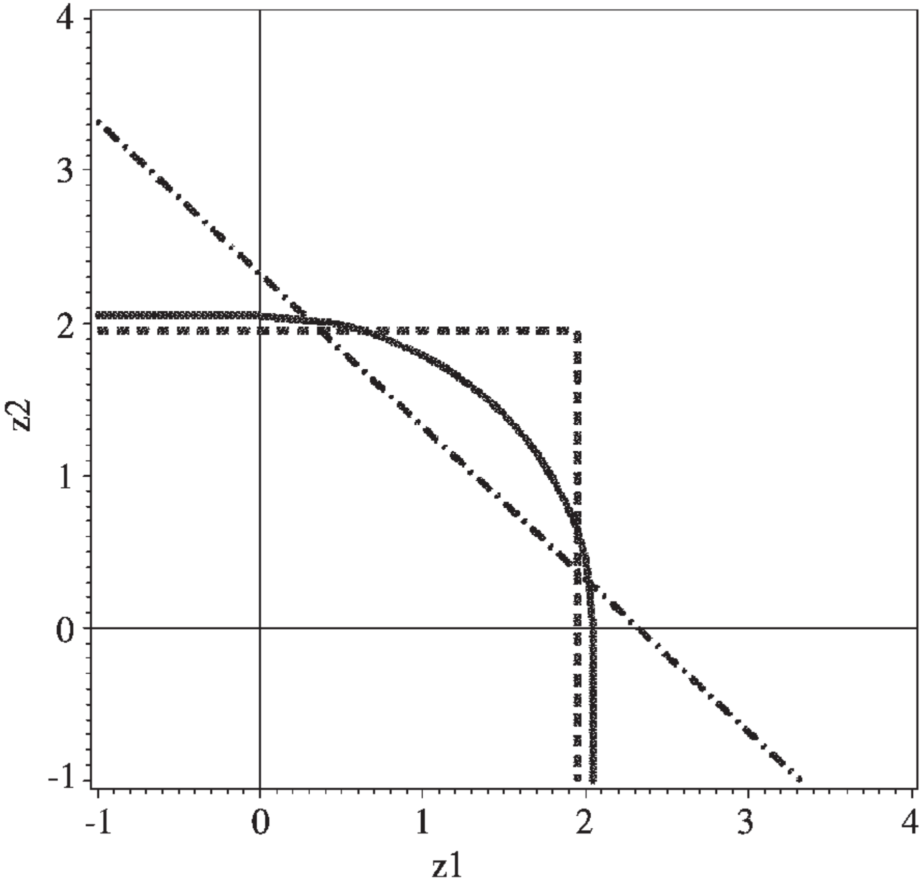
Acceptance / Rejection Regions

Example:

Figure 1 from Logan (2003).

Shows the decision of three tests depending on the observed value of the standardized mean vector.

Can only be used for the comparison of different tests based on some common multivariate statistic.



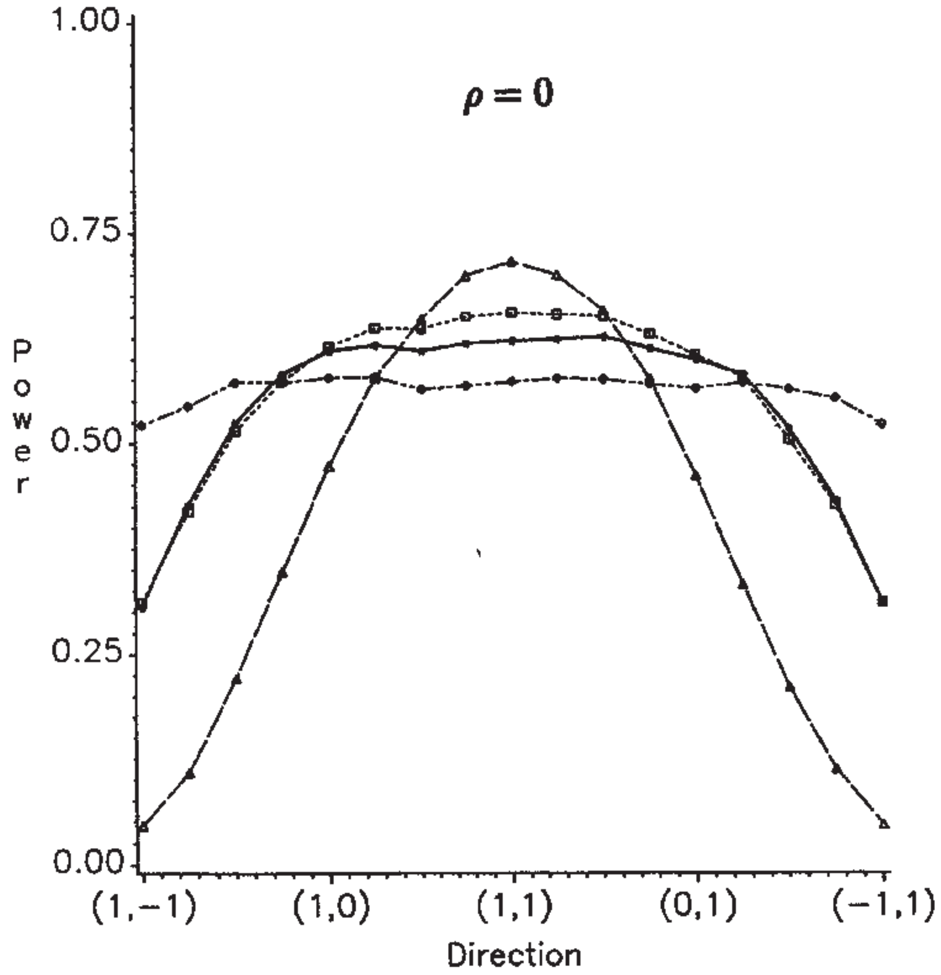
Power at a Fixed Distance from the Origin

Example:

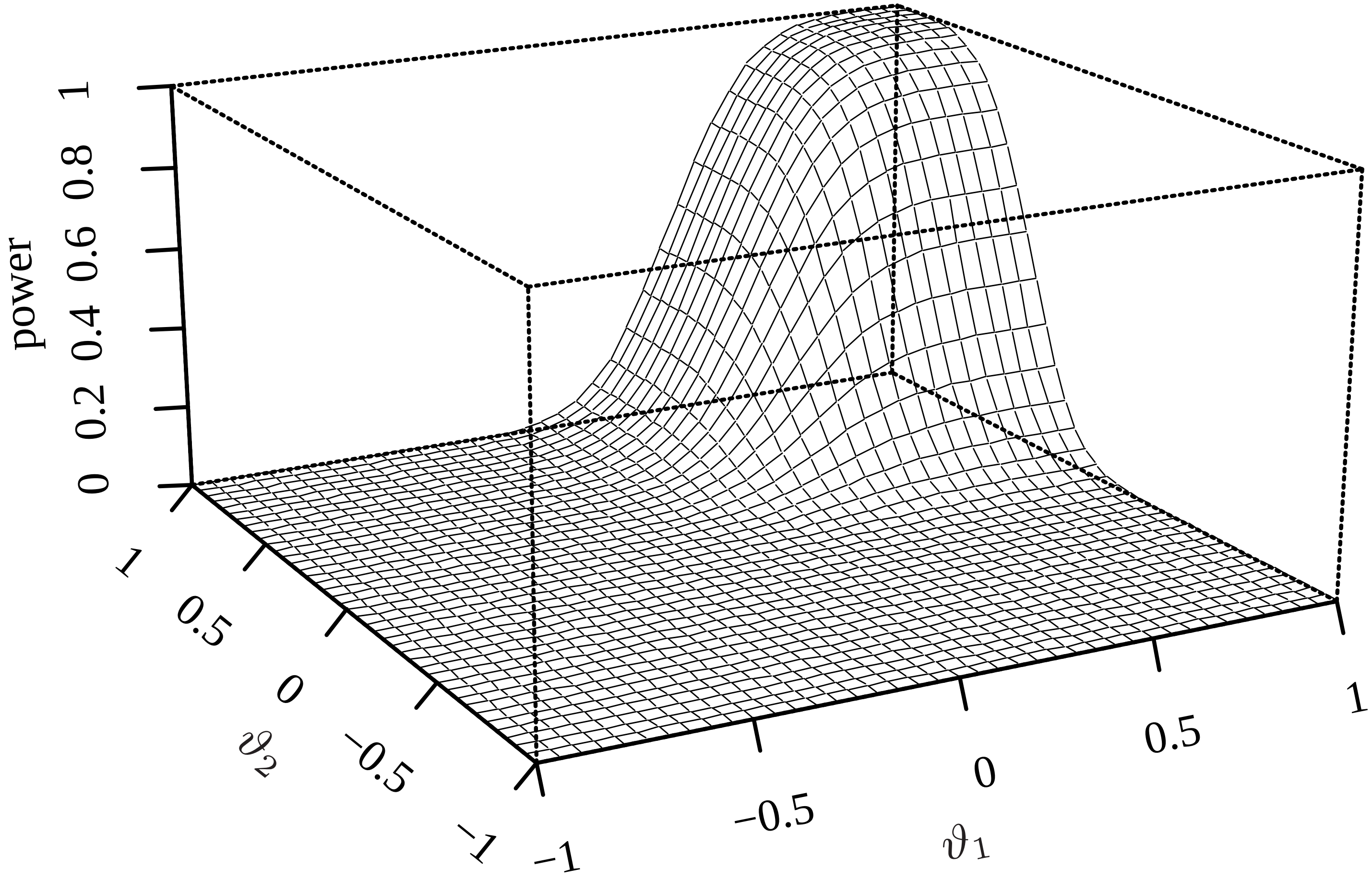
Part of Figure 2 from Follmann (1996).

Simulated power of four tests depending on the angle of the location parameter (constant distance from the origin).

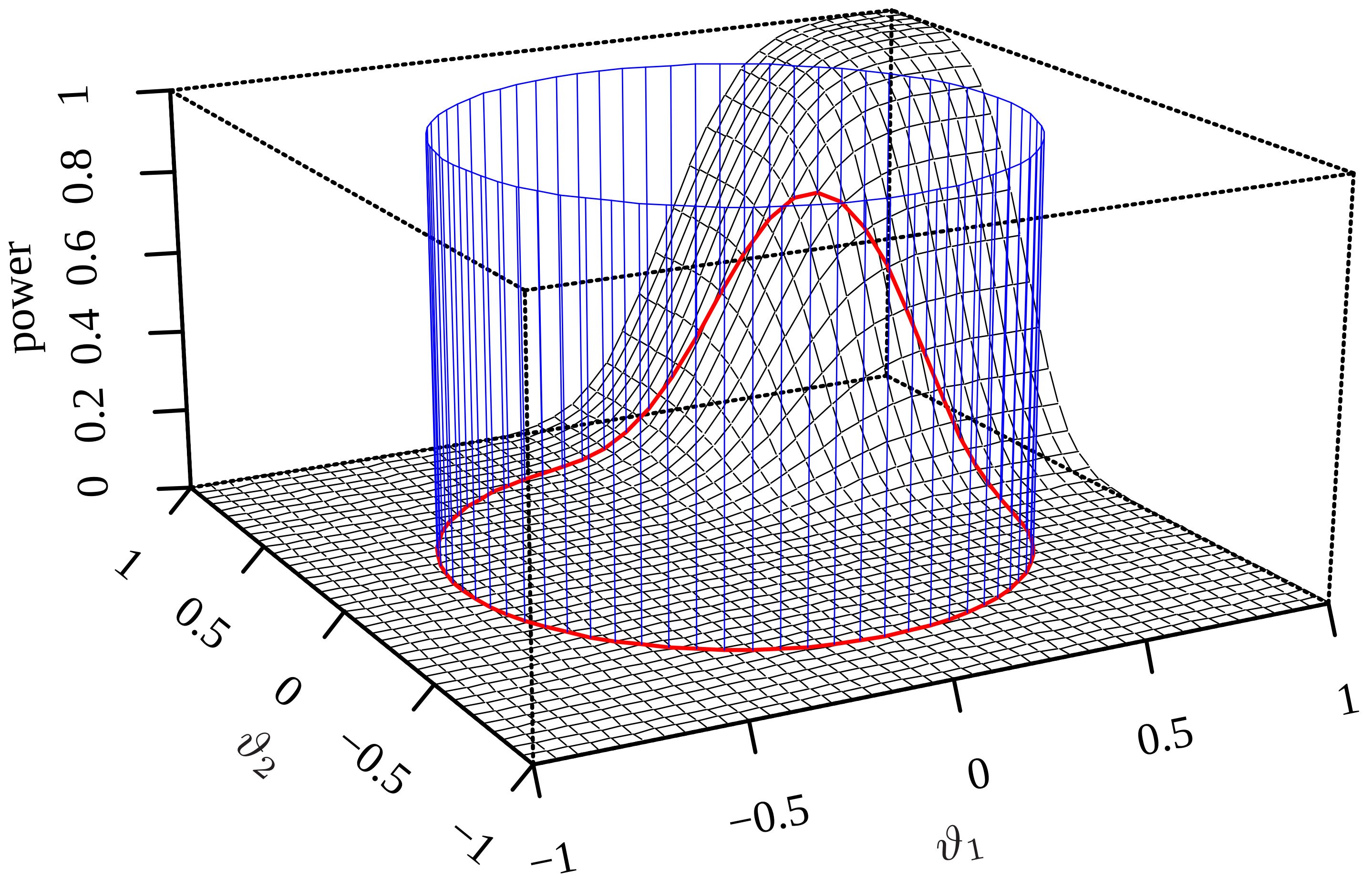
Depends on the distance from the origin; no information about the actual significance level at $\mathbf{0}$.



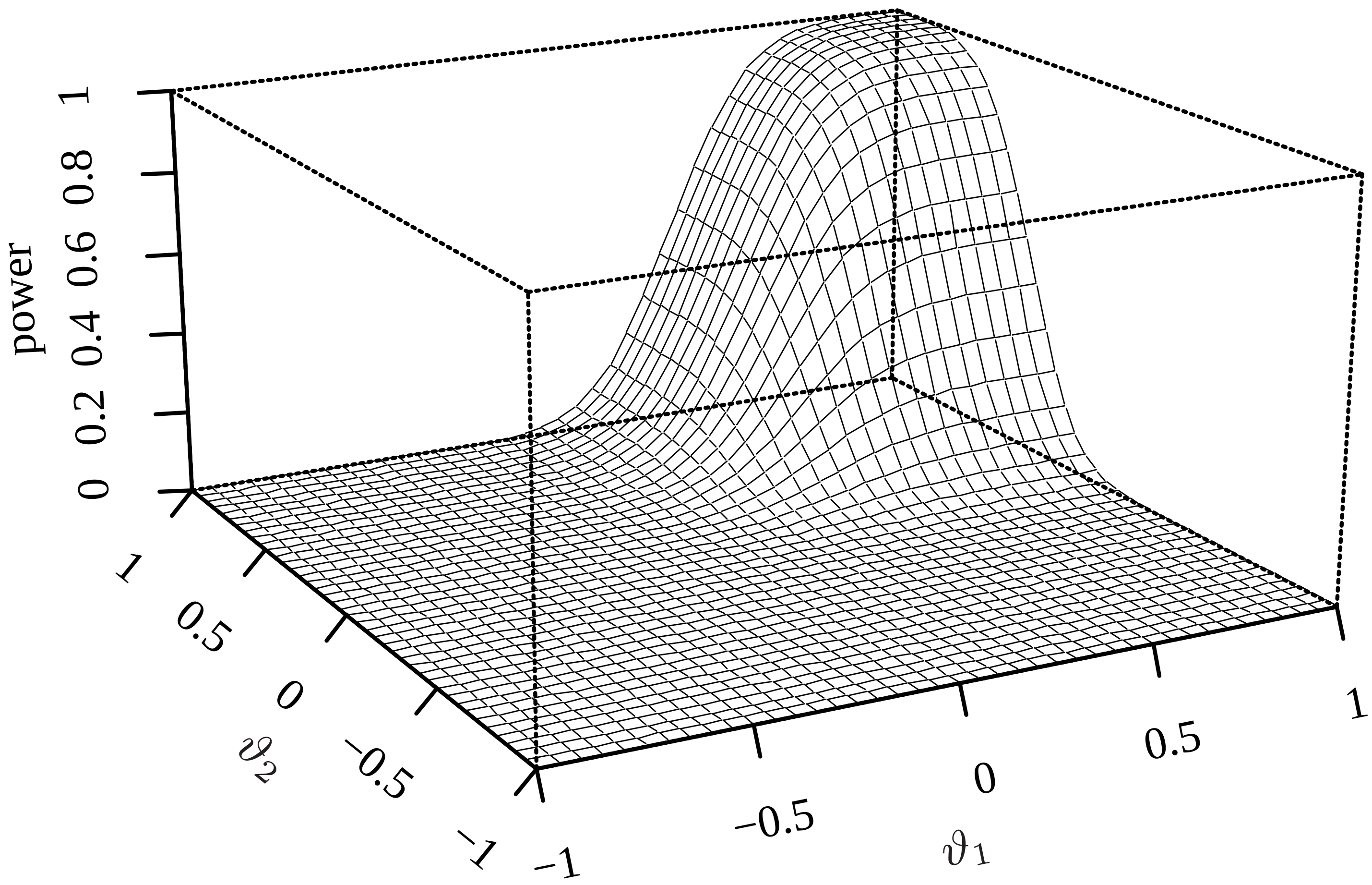
Power of the Sign Min Test $(\mathrm{N}(\mathbf{0}, \mathrm{I}_2),\ n = 50)$



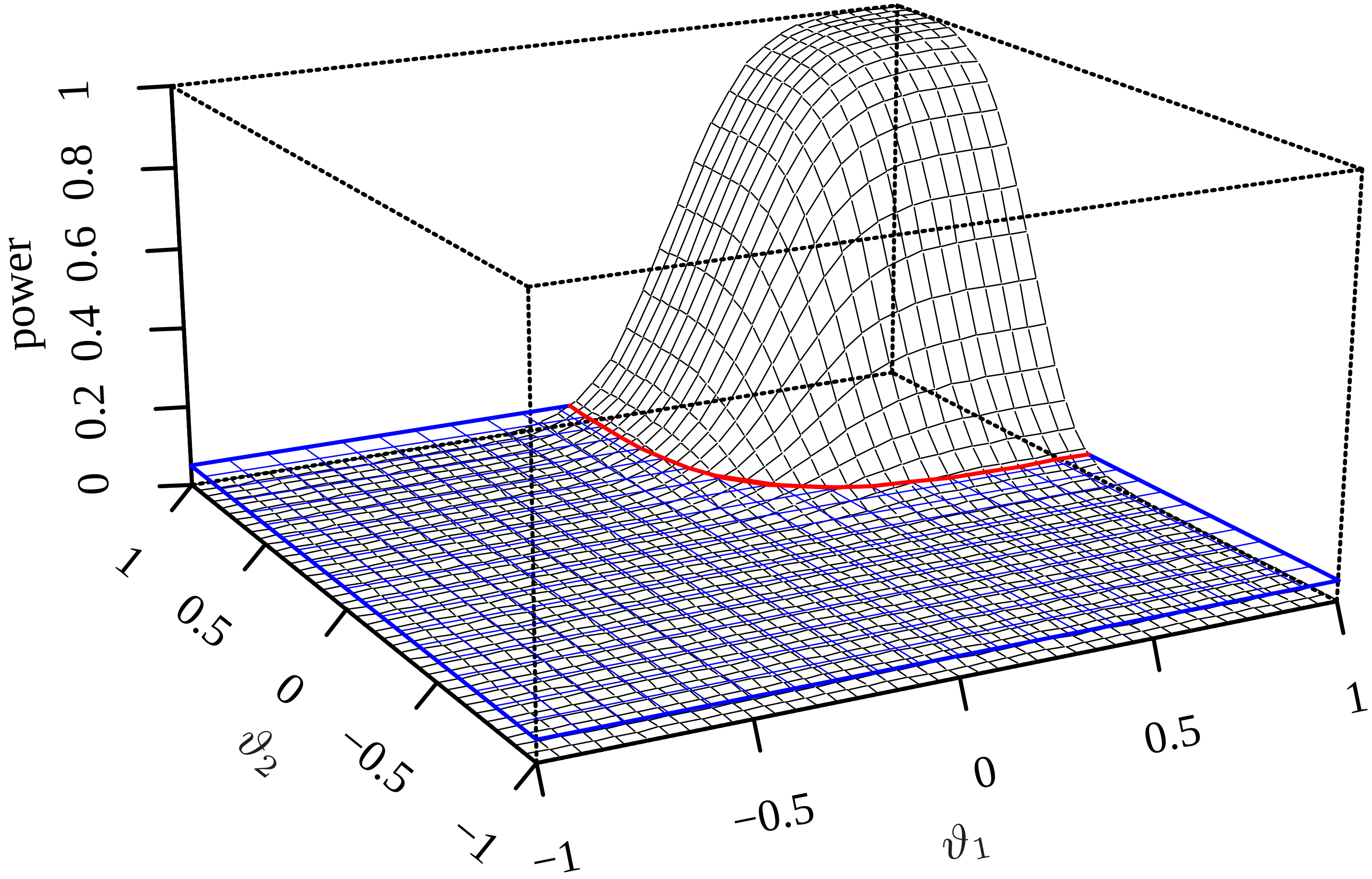
Intersection with a Cylinder



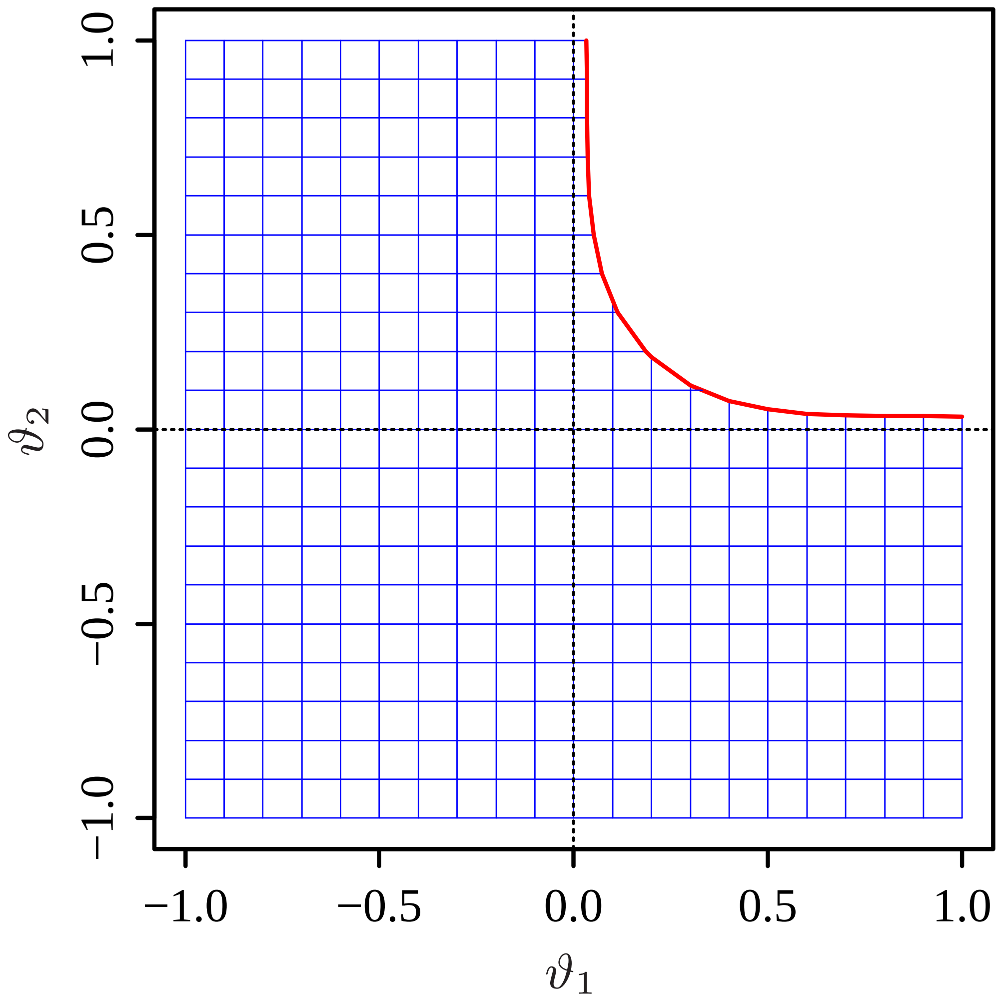
Power of the Sign Min Test $(\mathrm{N}(\mathbf{0}, \mathrm{I}_2),\ n = 50)$



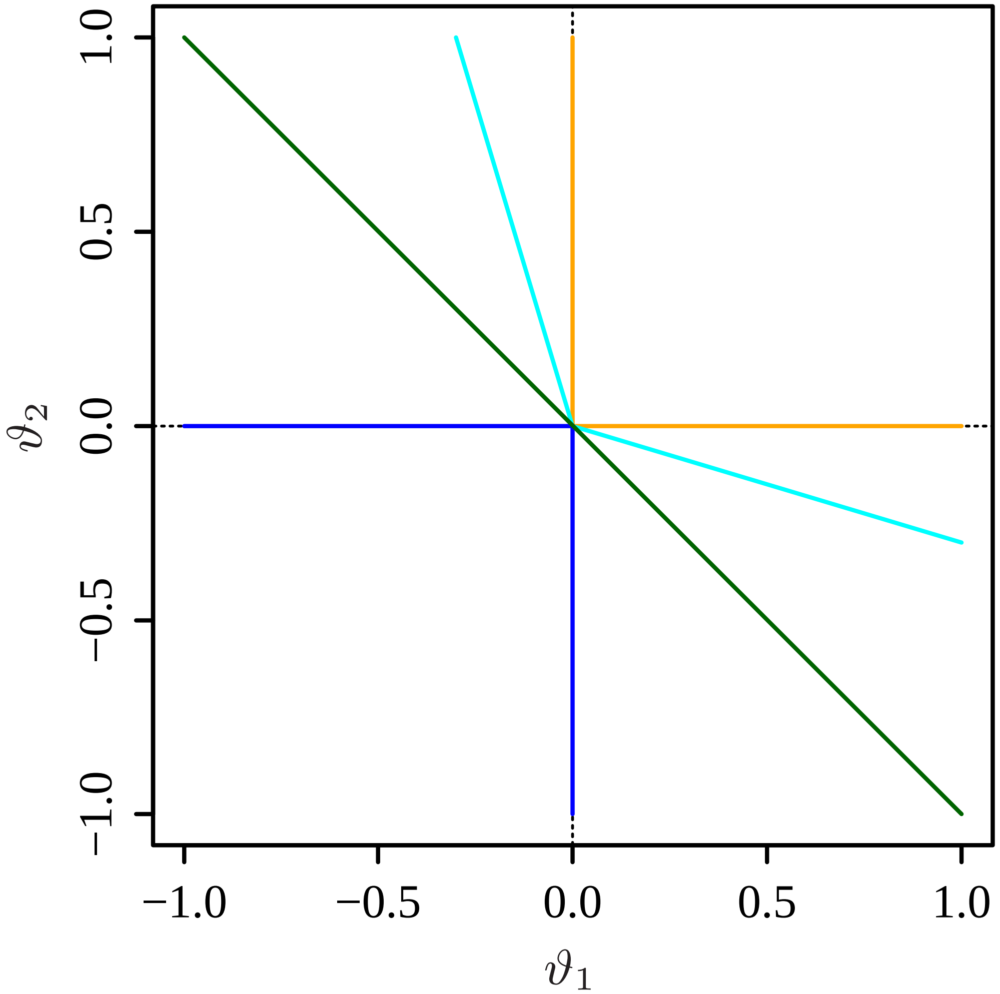
Horizontal Section at Power 0.05



Curve of Location Parameters that Lead to Power 0.05



Ideal Curves for Different Hypotheses



$H_0: \boldsymbol{\vartheta} \in \mathbb{R}^p \setminus \Theta_1$ vs.

$H_1: \boldsymbol{\vartheta} \in \Theta_1$

- $\Theta_1 =$ positive quadrant
- $\Theta_1 =$ convex sector
- $\Theta_1 =$ half plane
- $\Theta_1 =$ complement of the negative quadrant

Simulation: Tests Used

- Sign min test

  Reject H_0 if both univariate sign tests reject $H_{0i} : \vartheta_i \leq 0$.

- Wilcoxon min test

  Reject H_0 if both univariate Wilcoxon signed rank tests reject $H_{0i} : \vartheta_i \leq 0$.

- Larocque/Labarre (2004)

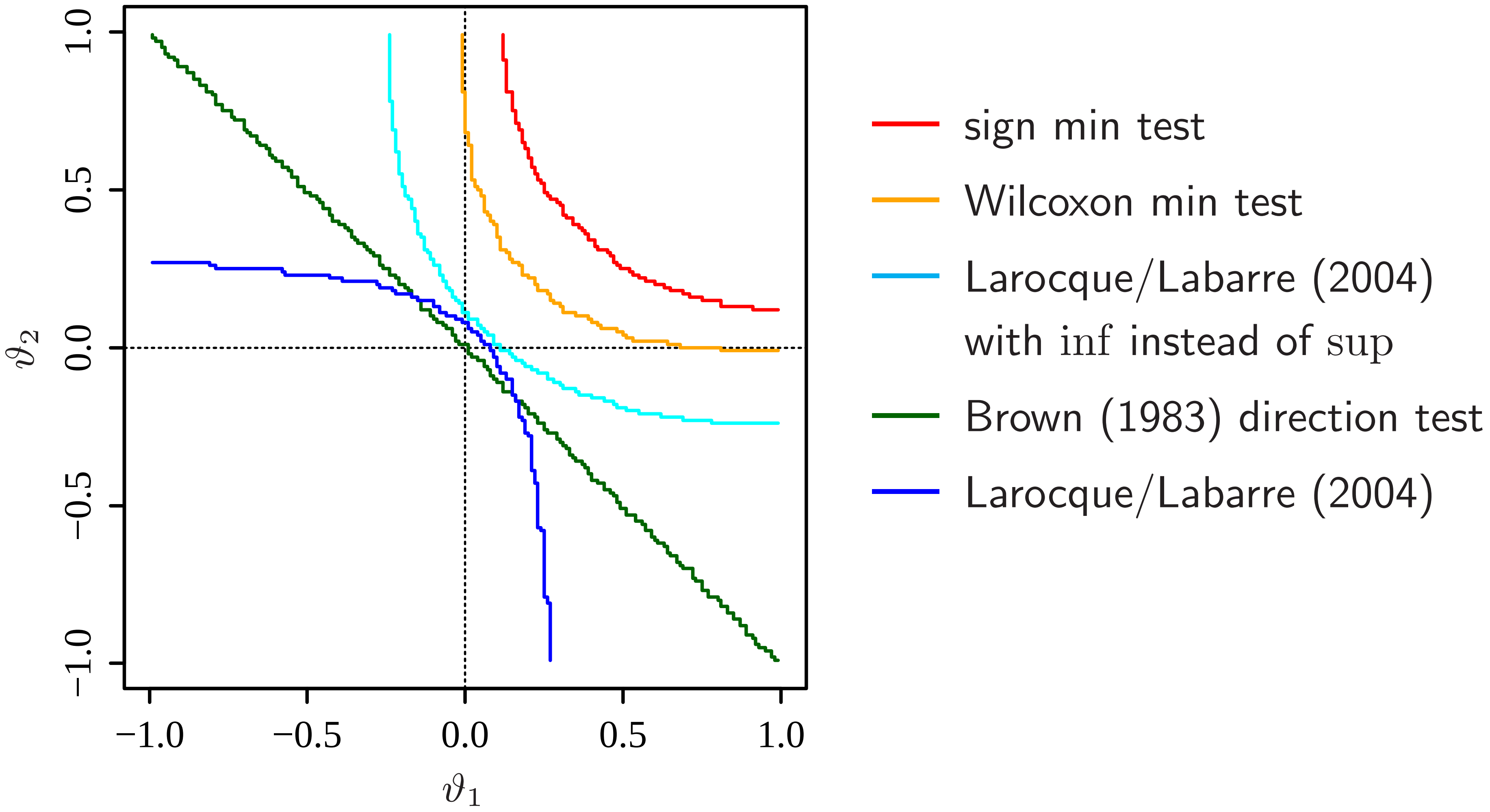
  Statistic: $\sup_{\|\boldsymbol{a}\|=1, \boldsymbol{a} \geq \boldsymbol{0}} \# \left\{ i : \boldsymbol{a}^{\mathrm{T}} \boldsymbol{X}_i > 0 \right\}$

- Larocque/Labarre (2004) with $\inf$ instead of $\sup$

- Brown (1983) direction test

  Statistic: $\sum_{i=1}^{n} \cos \left(\psi_i - \frac{\pi}{4} \right)$

Simulation: $\mathrm{N}(\mathbf{0}, \mathrm{I}_2)$, $n = 20$



Conclusion: Applications

Uses of curves of parameter values that lead to some fixed power:

- controlling the nominal level of a test
- determining the actual (composite) null hypothesis
- classification and comparison of (parametric or nonparametric) tests with respect to their hypotheses
- power comparisons (using several power values)

References

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