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1.0 INTRODUCTION
A graph is usually visualized by representing each vertex through a point in the plane, and by representing each through a curve in the plane, connecting the points corresponding to the end vertices of the edge. Such a representation is called a drawing of the graph if no two vertices are represented by the same point, if the curve representing one edge does not include any other points, representing a vertex (except its end points), and if two distinct edges have at most one common point. A drawing of a graph is plane if no two distance edge intersect part form their end points. A graph is planar if it admits a plane drawing.

In electronic circuits, components are joined by means of conducting strips. These may not cross, since this would lead to undesirable signals. In this case, an insulated wire must be used. For layers without crossings, which are then pasted together? The good is to use a few layers are possible. In this application it would be desirable to know the thickness of a hyper graph whose nodes are cell to be paced and whose hyper edges correspond to the nets connecting the cells. If the thickness problems could be should for graphs, it would be a useful engineering tool in the layout of electronic circuits.

2.0 THEORETICAL BOUNDS
Theorem (2.1) by Harary (1991).
let G' = (V, E') be a maximum planer sub graph of a graph G = (V, E) them |E'| ≤ 3|V|−6.

Theorem (2.2) by Harary (1991).
let G' = (V, E') be a maximum planer sub graph of a graph G = (V, E) which does not contain any triangles then |E'| ≤ 2|V|−4.

Theorem (2.3) by Cimikowski (1994).
let m<9 then G is planer.

Theorem (2.4) by Cimikowski (1994).
if G is planar and n≥4 then it has at least four vertices of degree<6
The problem of determining a maximal planar sub graph is NP-complete.

Theorem (3.5) by Clia and Goldmacher (1977), Yannakakis (1978) and Sylslo (1978)
MPS is NP-complete.

Theorem (2.6) by Kotzig (1955)
the maximum planar sub graph of Qₙ contains 2ⁿ⁺¹⁻⁴ edges.
3.0 ALGORITHM FOR MPS

In 3.1 the performance ratio $R_A(p)$ of an approximation algorithm $A$ for a maximization problem $P$ is the minimum ratio of obtained solutions to the cost of optimal solution:

$$R_A = \begin{cases} \min \frac{A(G)}{OPT(G)}, & \text{if } OPT(G) \neq 0 \\ 1, & \text{if } OPT(G) = 0 \end{cases}$$

Since a spanning tree contains $n-1$ edges, and a maximum planar subgraph could contain at most edges (by theorem (2.1)) the performance ratio of this method is $1/3$.

$$\lim_{n \to \infty} \frac{n-1}{3n-6} = \frac{1}{3}$$

And since a maximal planar subgraph of any graph $G$ can have more than $2n-4$ (if $G$ is triangle-free), and any spanning tree of $G$, which is bipartite, has $n-1$ edges the performance ratio of this method is $1/2$. (By Theorem (2.2))

$$\lim_{n \to \infty} \frac{n-1}{2n-4} = \frac{1}{2}$$

3.1 GRE for MPS

A greedy algorithm to search a maximal planar subgraph is to apply a planarity testing algorithm and to add as many edges as possible to a planar subgraph. See algorithm (3.1) (GRE) for a detailed description of this edge adding method.

$$\text{GRE} (G= (V, E), G'=(V', E'))$$

(i) $E''=E\setminus E'$;  
(ii) while there is an edge $(u, v)$ is $E''$  
(iii) do $E' \leftarrow E' \cup \{(u, v)\}$, $E'' \leftarrow E'' \setminus \{(u, v)\}$  
(iv) if $(V, E')$ is not planar  
(v) then $E' \leftarrow E' \setminus \{(u, v)\}$;  
(vi) return $(V, E')$.

4.0 SIMULATED ANNEALING, SA

SA algorithm imitated the cooling process of material in a heat bath. SA was originally proposed by Kirkpatrick et al. (1983) based on some idea given by Metropolis et al. (1953). The search begins with initial temperature $t_0$ and ends when temperature is decreased to frozen temperature $t_f$, where $0 \leq t_f \leq t_0$.

The equilibrium detection rate $r$ tells when an equilibrium state is achieved, and temperature current temperature by the cooling ratio $\alpha$, where $0<\alpha<1$. To determine good parameters for a given problem is often a hard task, and it needs experimental analysis. There are also adaptive techniques for SA. (Van Laarhoven and Aarts, 1987).

The temperature of the SA algorithm gives the probability of choosing solutions that makes the current solution worse. If there are many bad solutions in a neighborhood of a "better" solution, and these bad solutions are accepted too often, the algorithm does not converge to good solution. For, more details concerning SA algorithm, see (Van Laarhoven and Aarts, 1987), (Metropolis et al., 1953) and the references given there;

Select a cooling ratio $\alpha$ and an initial temperature $t_0$;
Select a frozen temperature $t_f$ and an equilibrium detection $r$;
Select an initial solution; set $t \leftarrow t_0$ and $e \leftarrow 0$;

(i) while $t \geq t_f$ do  
(ii) while $e \leq r$ do  
(iii) $e \leftarrow e+1$  
(iv) randomly select a solution $s \in N(S_0)$  
(v) $\delta \leftarrow \text{cost}(s) - \text{cost}(S_0)$  
(vi) generate a random integer $i; 0 \leq i \leq 1$;  
(vii) if $i \leq e \cdot \delta/t$ then  
(viii)$S_0 \leftarrow s$
Now we introduce a SA algorithm for determining MPS. The algorithm gets as input a graph $G = (V, E)$ and a planar sub graph $G' = (V, E')$ of $G$. SA maintains two sets of edges. The first set, $E_1$, is initialized as $E'$. The second set, $E_2$, is initialized as $E/E'$. The first set maintains the edges of a maximum planar sub graph and the second set always contains the remaining edges of the input graph. In what follows, we say shortly that a set of edges is planar, if the graph induced by this edge set is planar. After initialization, local optimization guided by SA scheme is applied to increase the size of $E_1$ in the following way. An edge $e_2$ from $E_2$ is chosen randomly. First SA tries to move $e_2$ to $E_2$ without violating the planarity of $E_1$ (this increase the size of $E_1$) if this is not possible, SA randomly chooses and edge $e_1$ from $E_1$ and tries two swap this edge with $e_2$. If the planarity of $E_1$ is violated, SA checks if it is acceptable (according to the rules of SA) to move $e_1$ to $E_2$ (this decreases the size of $E_1$). To check that is it allowed to move an edge $e_1$ to $E_2$, we make a random test. A random real $i$, $0 \leq i \leq 1$, is generated if $i \leq e^{-1/t}$, where $t$ is the current temperature, holds then a move that decreases the size of $E_1$ is accepted. Since an $n$-vertex planar graph has at most $3n-6$ edges (by theorem (2.2)) we have added a test to recognize optimal solutions in the while-loops of SA.

4.1 SA for MPS

Select Cooling $\alpha$ and initial temperature $t_0$, select frozen ratio $\alpha$ temperature $t_1$, and equilibrium detection rate $t_e$; find a planar sub graph $G' = (V, E')$ of $G$ and set $E_1 = E'$

(i) $E_2 = E \setminus E_1$, $t = t_0$ and $e = 0$
(ii) while $t \geq t_1$ and $|E_1| < 3|V| - 6$ do
(iii) while $e \leq r$ and $|E_1| < 3|V| - 6$ do
(iv) $e = e + 1$
(v) randomly select an edge $e_2$ from $E_2$;
(vi) if $E_1 \cup \{e_2\}$ is planar then
(vii) set $E_1 = E_1 \cup \{e_2\}$ and $E_2 = E_2 \setminus \{e_2\}$;
else
(viii) randomly select an edge $e_1$ from $E_1$;
(ix) if $\{E_1 \setminus \{e_1\}\} \cup \{e_2\}$ is planar then
(x) set $E_1 = E_1 \setminus \{e_1\}$ and $E_2 = E_2 \cup \{e_2\}$;
(x) set $E_2 = E_2 \setminus \{e_2\}$ and $E_2 = E_2 \cup \{e_1\}$;
else
(xi) generate a random real number $i$, $0 \leq i \leq 1$;
(xii) if $i \leq e^{-1/t}$ then
(xiii) set $E_1 = E_1 \setminus \{e_1\}$ and $E_2 = E_2 \cup \{e_1\}$;
end;
(xiv) $t = \alpha t$;
(xv) $e = 0$;
end;
(xv) return $(V, E_1)$;
end

5.0 BRANCH-AND-BOUND

One method to solve a discrete and finite optimization problem is to generate all possible solution and then choose the best one of them. For NP-complete problem this exhaustive search method fails since the number of solutions is exponential in the size of the input. For example, given a graph $G = (V, E)$, there exist $2^{|E|}$ different sub graph containing all $|V|$ vertices. If the problem in question asks a sub graph of the given graph with some specific property, it is possible that all sub graphs need to be checked before the right one is found. Only small instance can be solved in this way.

Branch-and-bound is a method that can be used in the exhaustive search by recognizing partial solutions that can not lead to an optimal solution. For example, suppose that we know that the optimal solution for a maximization problem is at least $K$ (we have found a solution with cost $K$). If during the generation of the other solution candidates we recognize that the current solution can not be augmented to any solution with cost $\geq K$, we can stop generating these solution candidates and continue searching from more promising solution. The efficiency of the branch-and-bound techniques depended highly on the problem in question and the order in which the solution candidates are found. If the lower bound for the optimal solution is bad, there are little possibilities to reject any solutions. The worst case during time of branch-and-bound is still exponentially but often it decreases the computation time remarkably (Kotzig, 1955).
5.1 Branch-and-Bound for MPS

Since a planar graph can have no more than $3n-6$ edges, and since any graph with fewer than 9 edges is trivially planar, it suffices to generate and test only sub graphs satisfying $3n-9$ these size constraints and hence at most $\sum_{i=5}^{n} \binom{m}{3n-i}$ is number. Secondly, since a non-planar graph may have less than $3n-6$ edges, say, $3n-k$ where one need test at most $\sum_{i=k}^{n} \binom{m}{3n-i}$ sub graphs, which is still exponentially large.

4.0 CONCLUSION

In this paper, two algorithms concerning the maximal planar sub graph (MOPS) in particular branch-and-bound algorithm for solving the maximum planar sub graph problem have been presented techniques for reducing the number of sub graphs processed during the search have been described.

REFERENCES